Products based on regional modelling approaches for future satellite gravity missions

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Outline

- Context
- Review of a few regional modelling approaches
  - Characteristics
  - Examples of application
- Comparisons
- Conclusions

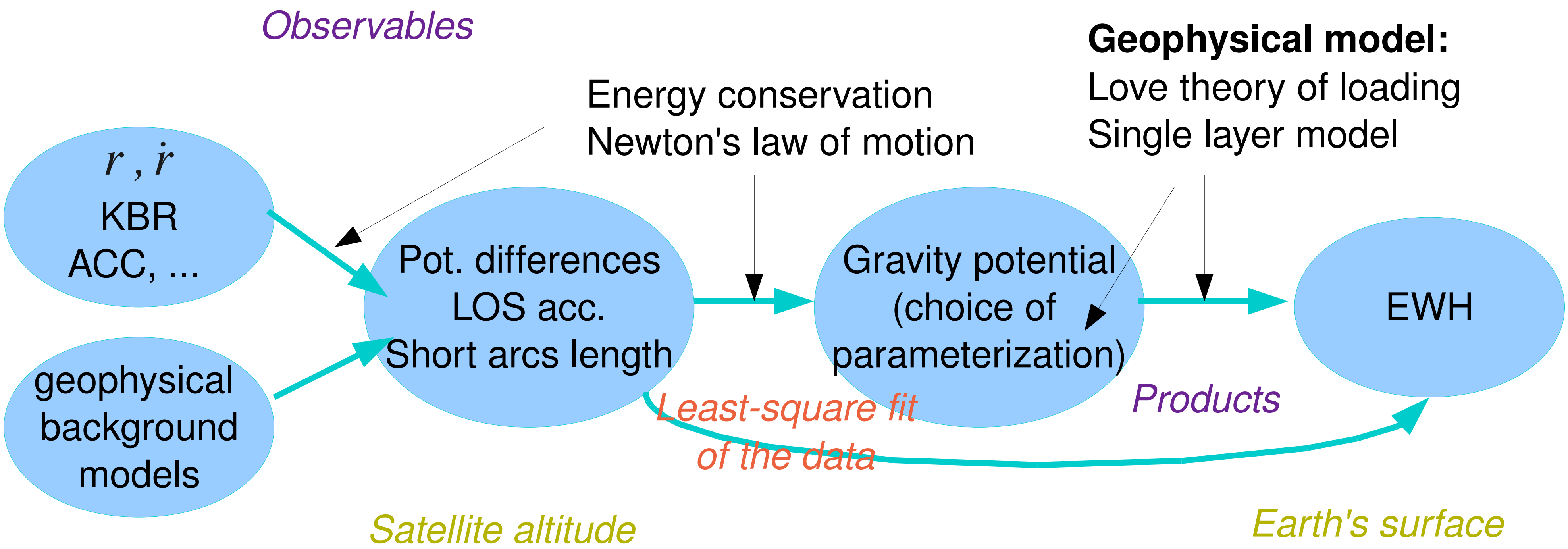
Context

- GRACE provides gravity data with global coverage, from which global models of the gravity potential (V) are computed
  - *L2 products: spherical harmonics (SH) expansions of V*
  - *'L3 products': equivalent water heights EWH are derived.*
- Interest of users for regional mass variations
  - *Enhance spatial resolution*
  - *Minimize leakage of mass variations outside the study area*
  - *Study area of various shapes (basins, …)*
  - *Combine GRACE mass variations with in-situ data*

➲ Development of localizing filters for SH solutions

➲ Development of regional modelling approaches

From the data to the model



- Linear(ized) model between the observables at satellite altitude and the gravity potential or EWH at the Earth's surface
- Least-squares inversion
  - *Ill-posedness due to the downward continuation ➲ Regularization*
  - *It is related to mission geometry (altitude, inter-satellite distance) and adequacy between data characteristics and field parameterization.*

Spherical harmonics

- SH expansion of the gravity potential V (central multipoles):

$$V = \frac{GM}{a} \sum_{\ell,m} \left(\frac{R}{a}\right)^{\ell} (C_{\ell,m} \cos m\phi + S_{\ell,m} \sin m\phi) P_{\ell,m}(\cos\theta)$$

- Each SH coefficient contains the integrated effect of all masses:

$$\begin{pmatrix} C_{\ell,m} \\ S_{\ell,m} \end{pmatrix} = \frac{3}{4\pi a \rho_m (2\ell+1)} \int \Delta\rho(r,\theta,\phi) P_{\ell,m}(\cos\theta) \left(\frac{r}{a}\right)^{\ell+2} \begin{pmatrix} C_{\ell,m} \\ S_{\ell,m} \end{pmatrix} \sin\theta d\theta d\phi dr$$

Density inside the whole Earth

- Surface load is derived using Love theory:

Surface load SH coefficients →

$$\begin{pmatrix} \hat{C}_{\ell,m} \\ \hat{S}_{\ell,m} \end{pmatrix} = \frac{\rho_m}{3\rho_w} \frac{2\ell+1}{1+k_\ell} \begin{pmatrix} C_{\ell,m} \\ S_{\ell,m} \end{pmatrix}$$

← *Gravity potential SH coefficients*

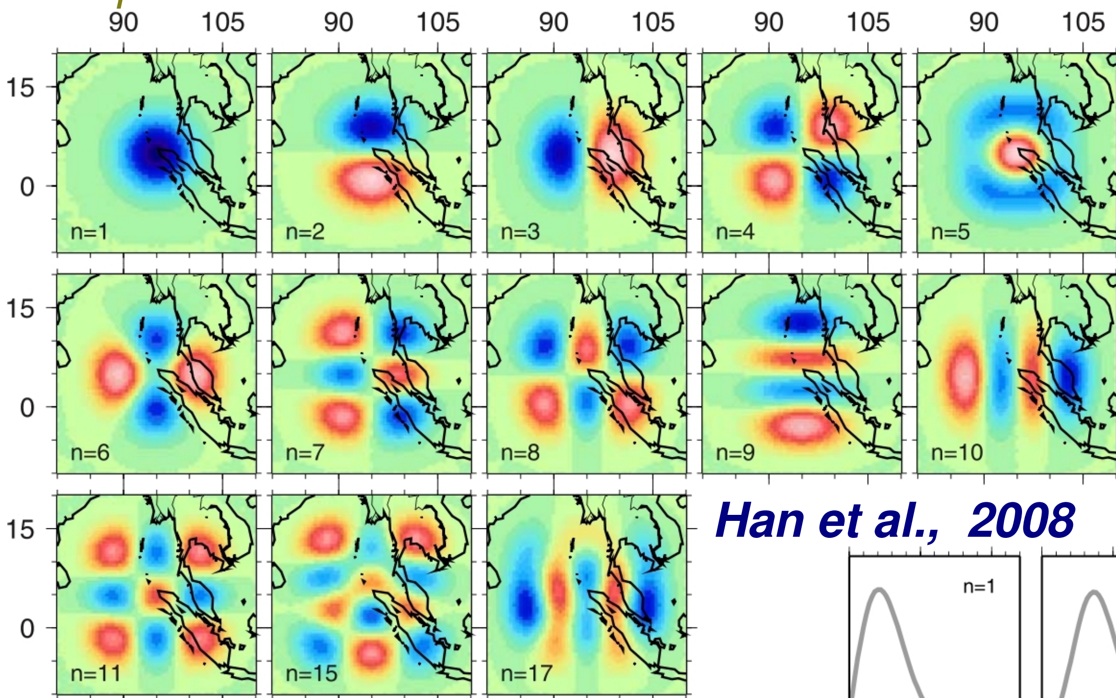
- Post-processing: basin localizing filters minimizing various error sources (Wahr *et al*., 1998, Swenson *et al*., 2003, and many others)

A few regional approaches

- **Increase flexibility** (tailored models) with less basis functions
- Computation of a harmonic gravity model
  - *Localized radial basis functions*
  - *The EWH is derived using loading Love numbers*
- Computation of a EWH model (single layer model)
  - *Mascons, related through their SH expansion to the satellite measurements (loading included)*
  - *'Mascons', directly related to the satellite measurements (no loading)*
    - *Also called 'regional approach'*
- *Our review of possible approaches is not exhaustive !!*

1. Slepian functions

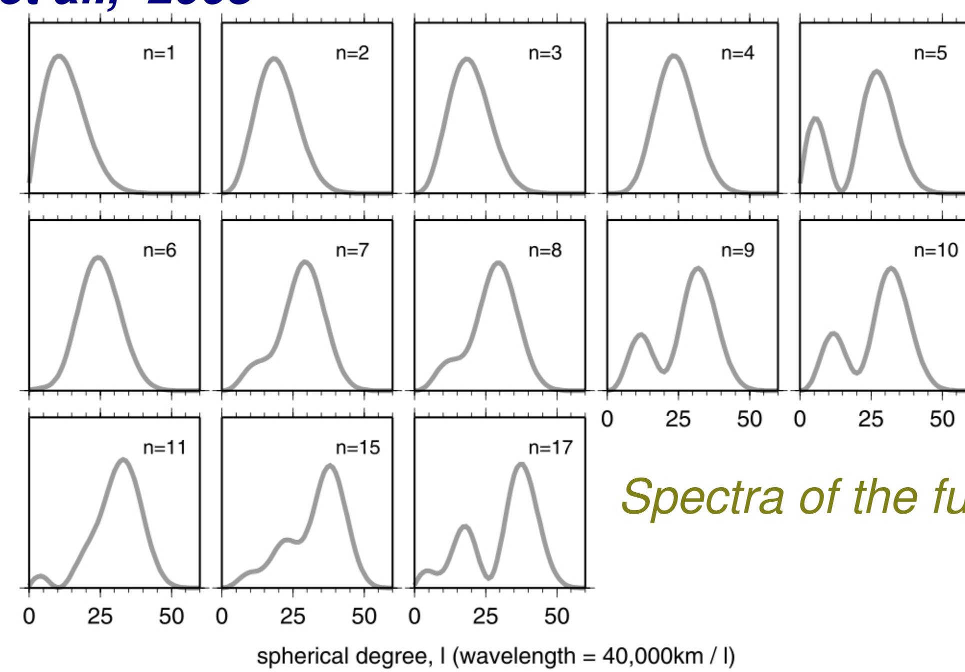
Slepian functions in the Sumatra subduction zone



- Orthogonal functions
- Optimal spatio-spectral energy concentration
- Uniform resolution over the area

  See also: Simons & Dahlen, 2006, Albertella et al., 1999.

Han et al., 2008



Spectra of the functions

- Applications:
  - Data modelling with polar gap
  - Regional contributions to global spectra
  - Regional field modelling

2. Splines

- No longer orthogonal
- Defined from the a-priori variance of the modelled signal

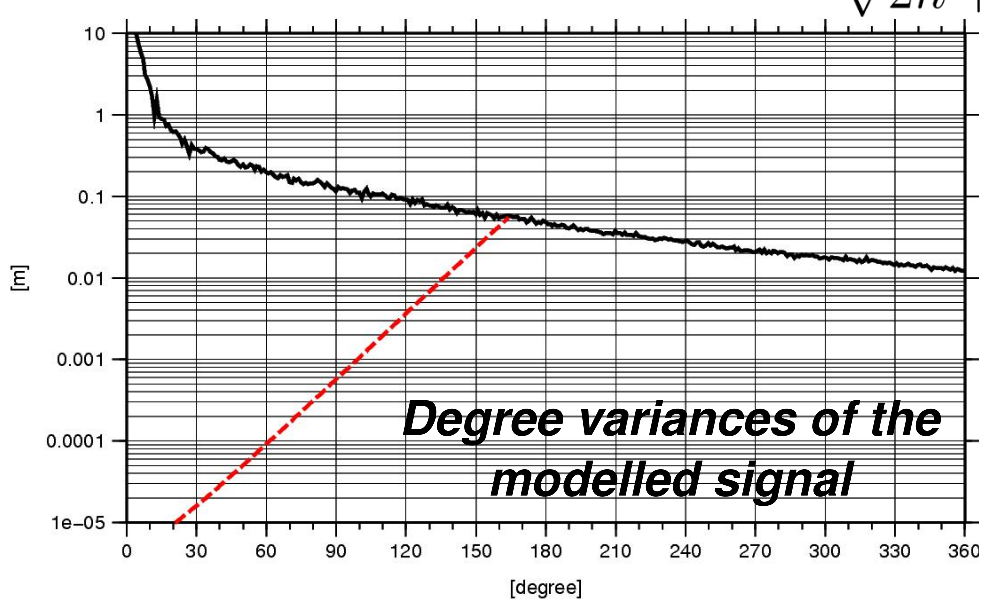
$$\Phi(\mathbf{x}, \mathbf{x}_i) = \sum_{n=2}^{\infty} \sum_{m=-n}^{n} k_n Y_{nm}(\mathbf{x}) Y_{nm}(\mathbf{x}_i)$$

Grid points

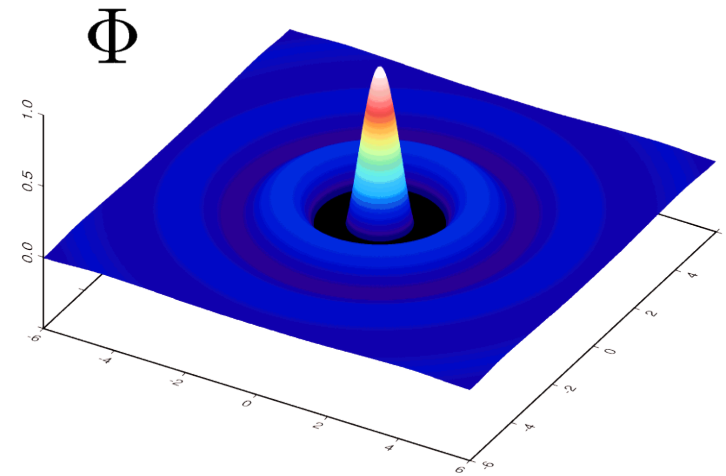
Shape coefficients

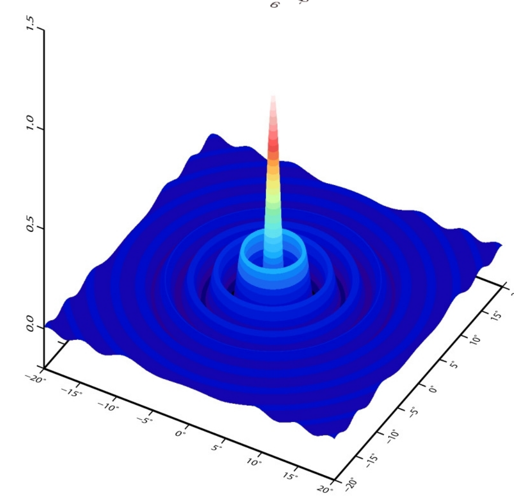
Spherical harmonics

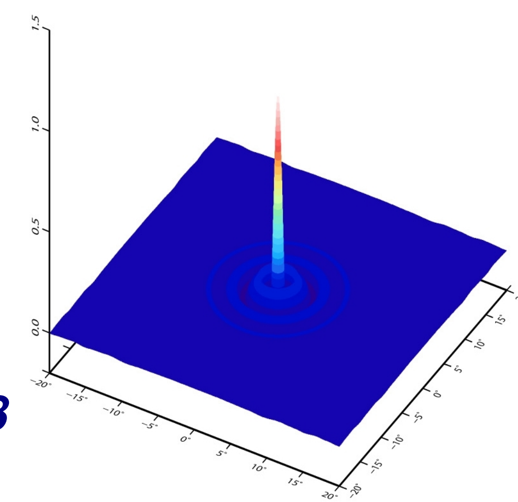
Choice of shape coefficients: $k_n = \dfrac{\sigma_n}{\sqrt{2n+1}}$



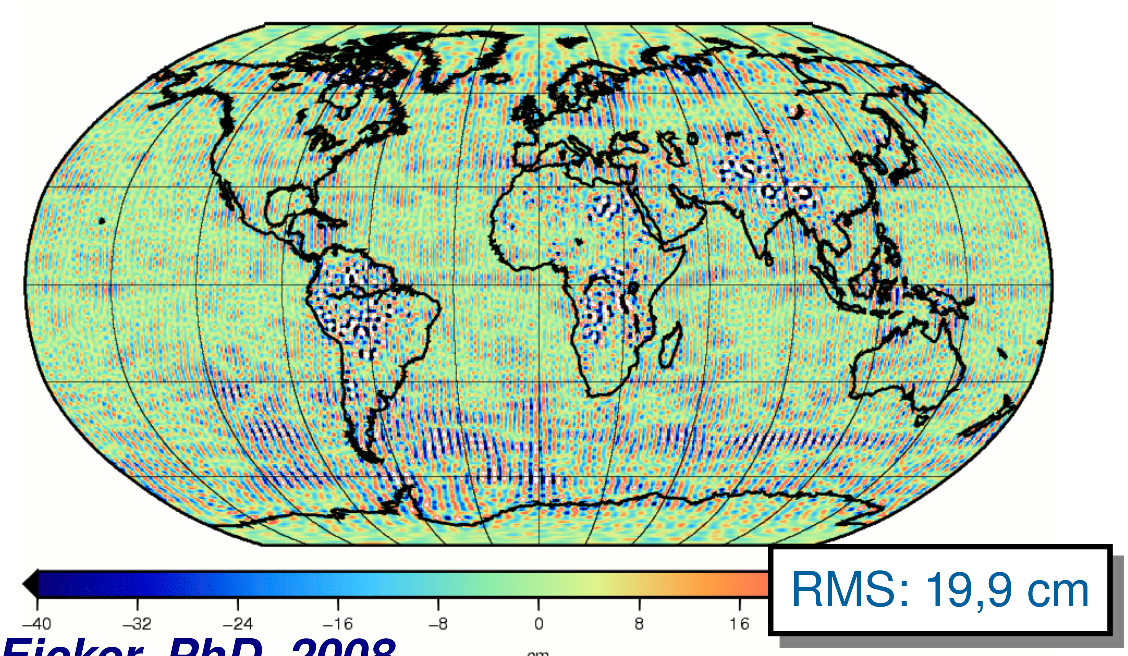
Eicker, PhD, 2008



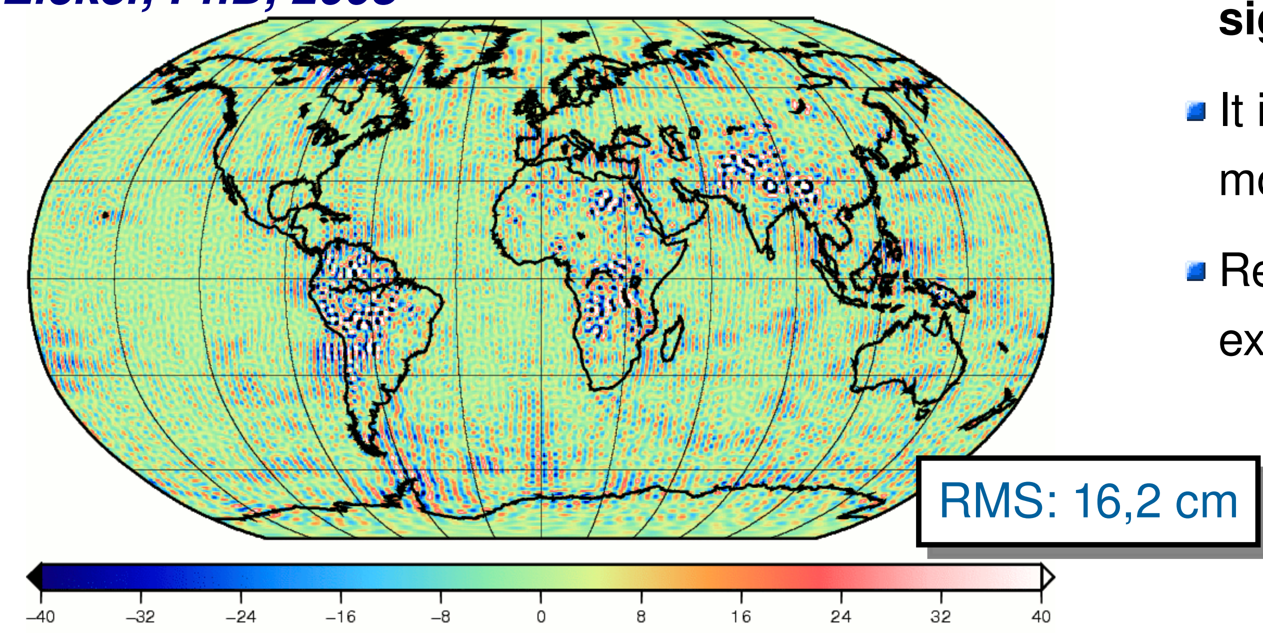




Application: local refinements from GRACE



Eicker, PhD, 2008



- Low frequencies from a global spherical harmonics model
- Residual signal is modelled with splines: **hybrid SH/spline models**.
- Spline model is computed by blocks, regionally
- **Tailored regularization and data selection for each block: increase signal-to-noise ratio**
- It is possible to gather the local spline models into a global one
- Reduced striping and enhanced signal extraction from the data.

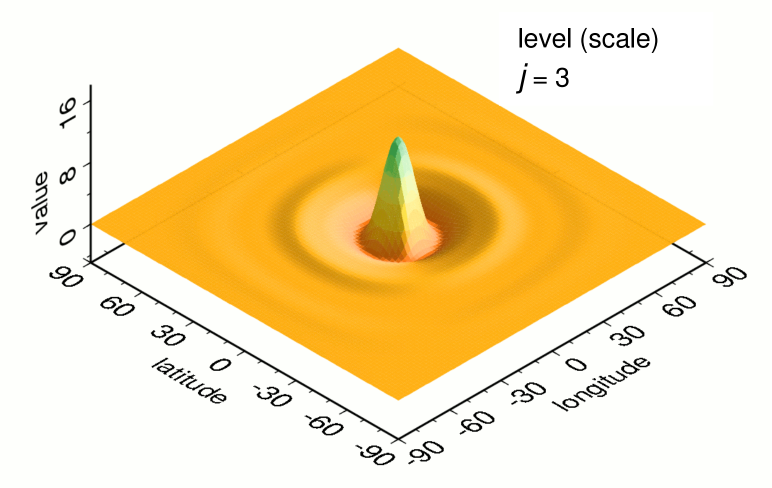
Top: ITG-Grace03s-EGM2008
Bottom: ITG-GraceSpline03-EGM2008.

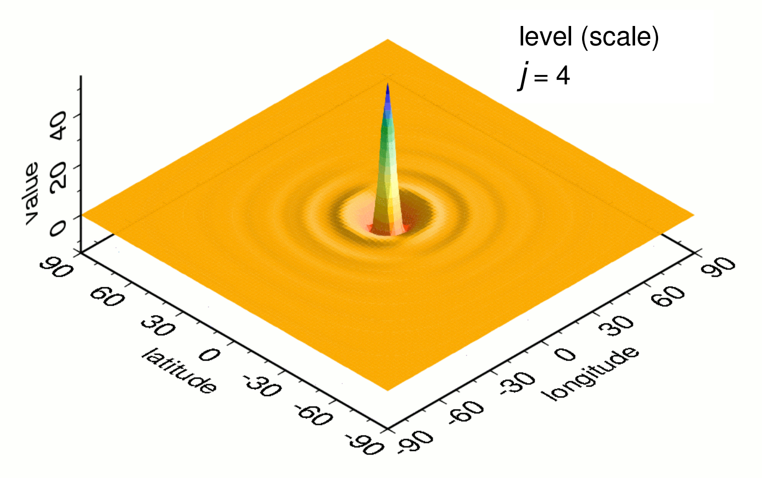
3.1 Multi-resolution approaches (3D): wavelets

- Non orthogonal, possibly redundant systems
- Principle: split a signal into a sum of details at different resolution levels (frequency bands).
- Each level of detail of the signal is represented by a linear combination of spherical wavelets at the corresponding scale.
- Wavelets at different scales and positions, well localized in space and frequency
- Adding all details of levels $i \leq i_{max}$ yields an approximation of the signal at level i_{max}. This approximation may also be written as a linear combination of scaling functions of level i_{max}.
- A wavelet can be written as the difference between two scaling functions at two consecutive levels of approximation: it is the detail to add to a coarser approximation in order to get a finer approximation of the signal.

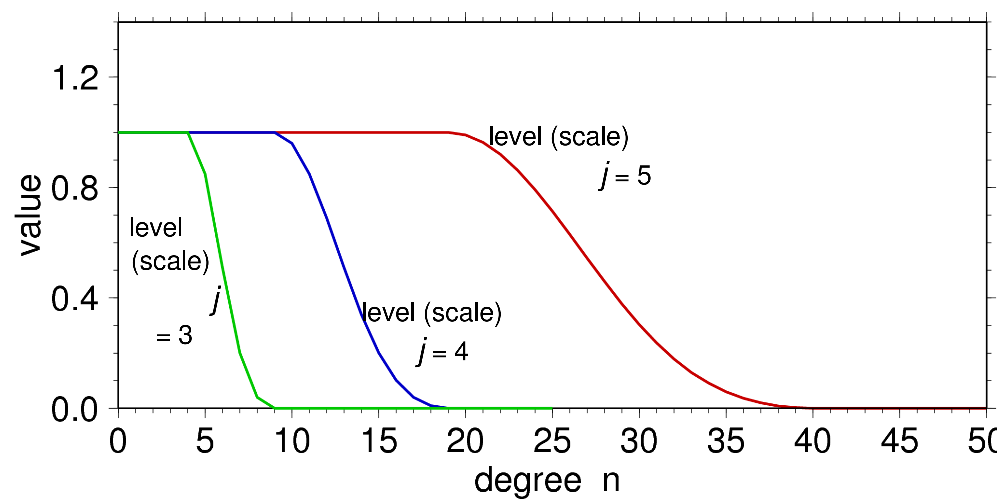
Blackman wavelets and scaling functions

spatial domain





spectral domain



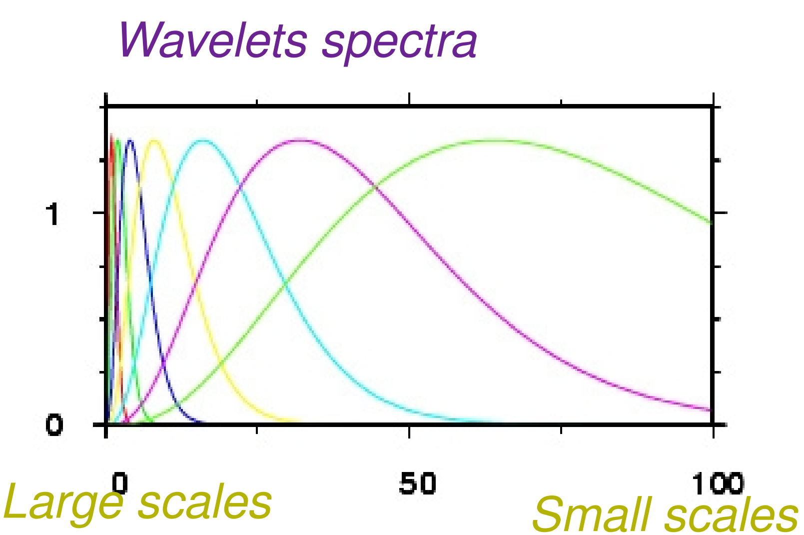
- The figure shows the frequency behavior of the **Blackman scaling function** for levels $j = 3,4$ and 5.
- The **lower** the level the **smoother** is the filtered signal.

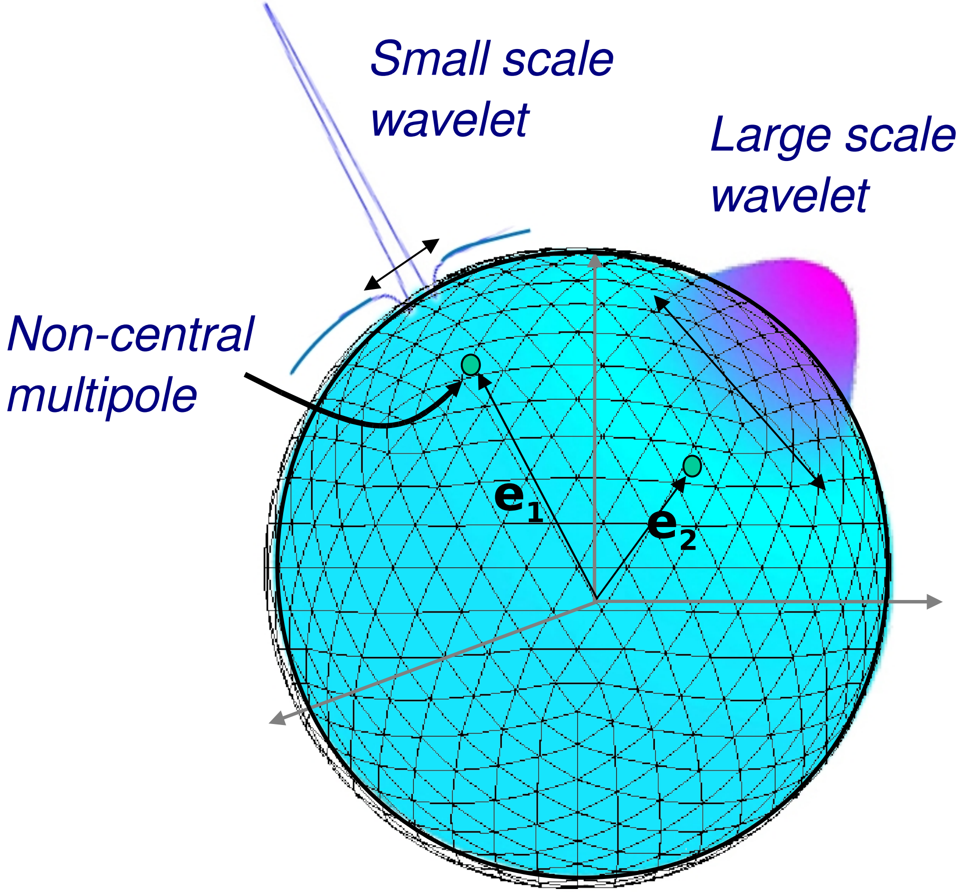
Schmidt et al., J. Geod., 2007

See also: Freeden et al., 1998

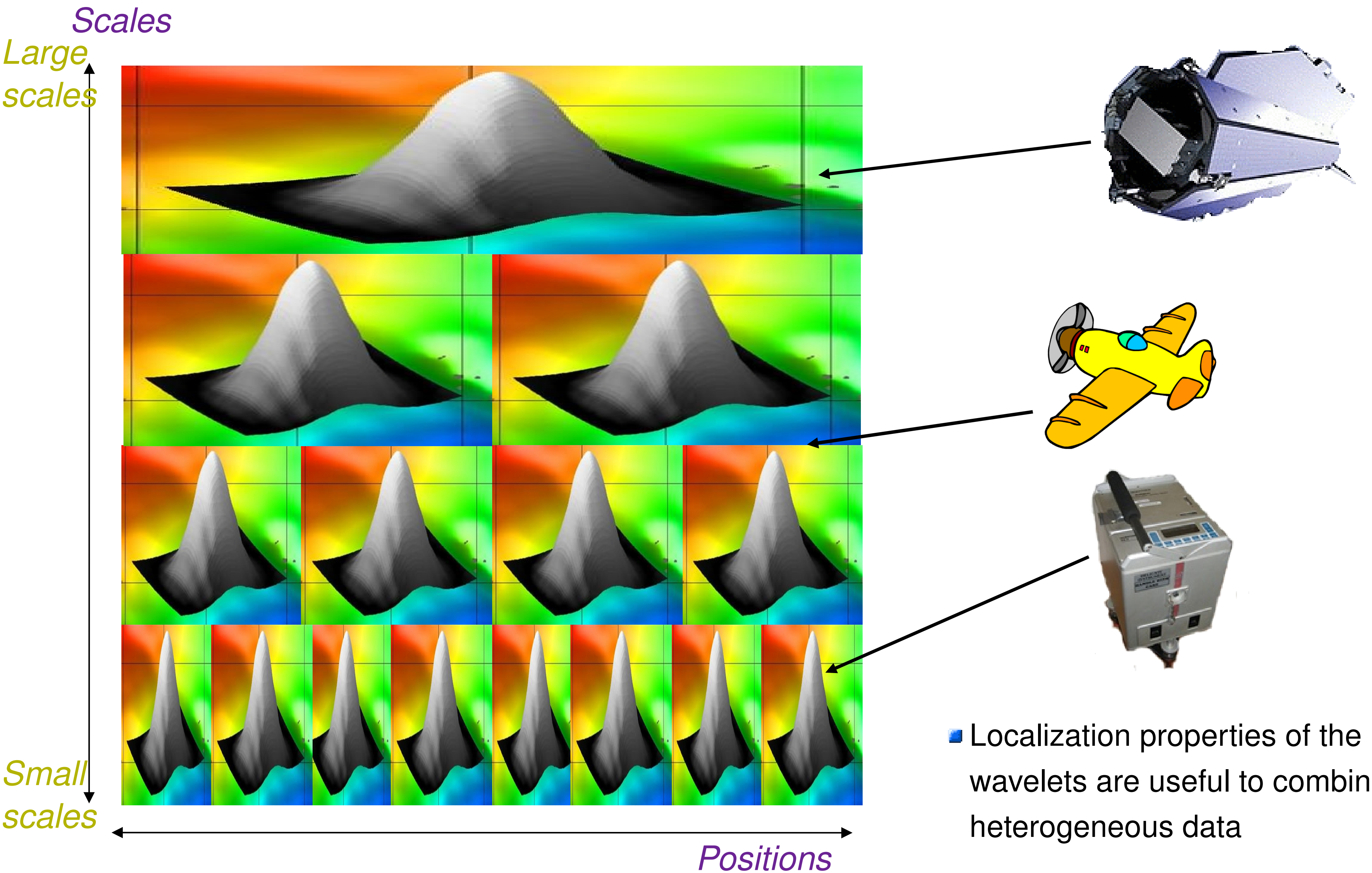
Poisson multipole wavelets

- Analytical formulation in terms of multipoles of low order (*Holschneider et al., 2003*): equivalent sources at depth.
- No scaling functions
- Scale sequence: dyadic.
- Positions at the vertices of spherical meshes. The finer the scales, the denser the mesh.



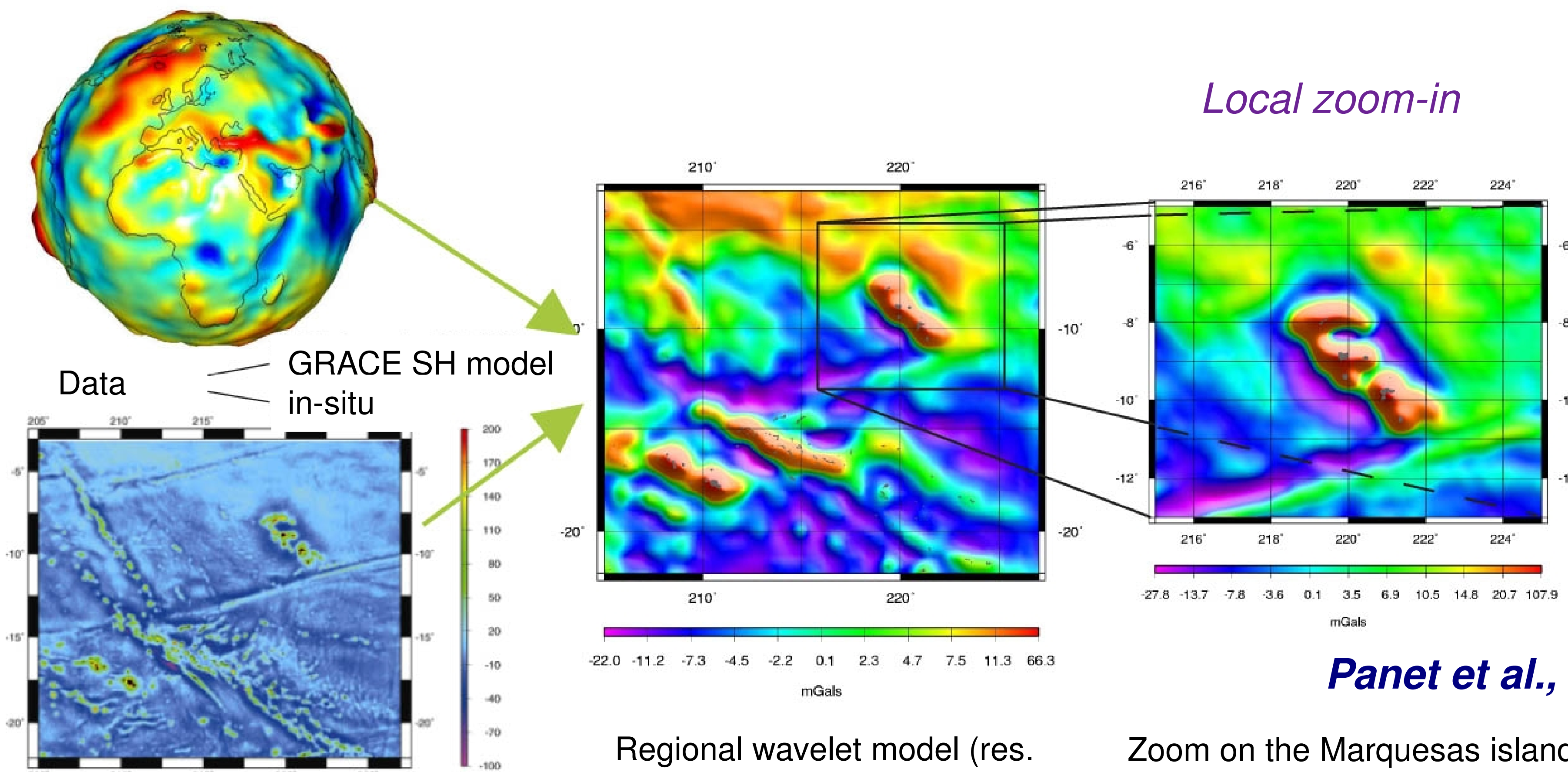


Application: local data combination



- Localization properties of the wavelets are useful to combine heterogeneous data

Example: local refinement of a global model



Panet et al., 2006

Regional wavelet model (res. 75 km, 9600 wavelets)

Zoom on the Marquesas islands (res. 30 km, 9500 wavelets)

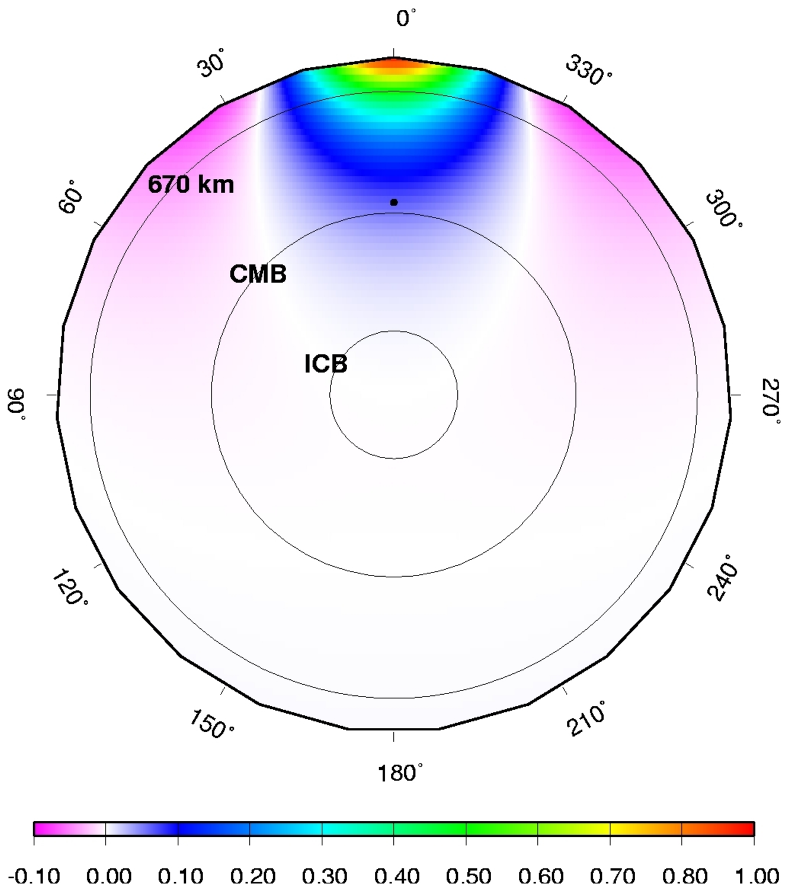
- Local refinements using high-resolution datasets: **hybrid SH/wavelet models**
- Scale/Block computations (Panet *et al.*, 2009) ➲ **tailored regularization & data weighting**
- Increase resolution over area of interest: zoom-in.

A multi-scale analysis of the modelled field

- The wavelet models of the gravity potential thus obtained also lead to a multi-scale analysis of the gravity potential

  ➲ *useful for geophysical studies and sources extraction*

- The correlations between the wavelets and the gravity potential provide an integrated, regionalized view of the densities



Weighting function of the densities

Panet et al., 2006

3.2 Extension to 4D

- The spatial multi-resolution may be coupled with a temporal one.
- Trade-off between spatial and temporal resolution, **set different temporal resolutions for the components at different spatial scales**.

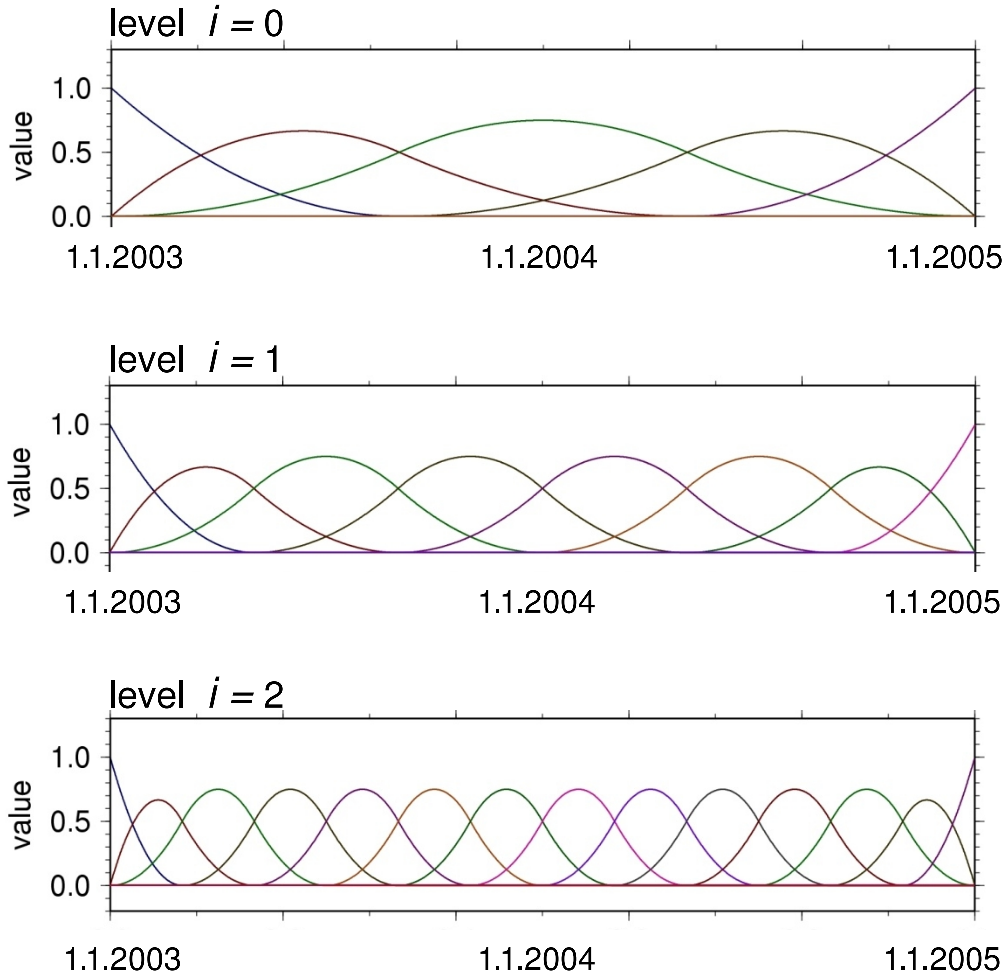
- In our geopotential representation

$$V(\boldsymbol{r},t) = \sum_{k=1}^{N_j} d_{j,k}(t)\ \phi_j(\boldsymbol{r},\boldsymbol{r}_k)$$

we model the time-dependent **scaling coefficients** $d_{j,k}(t)$ by the expansion

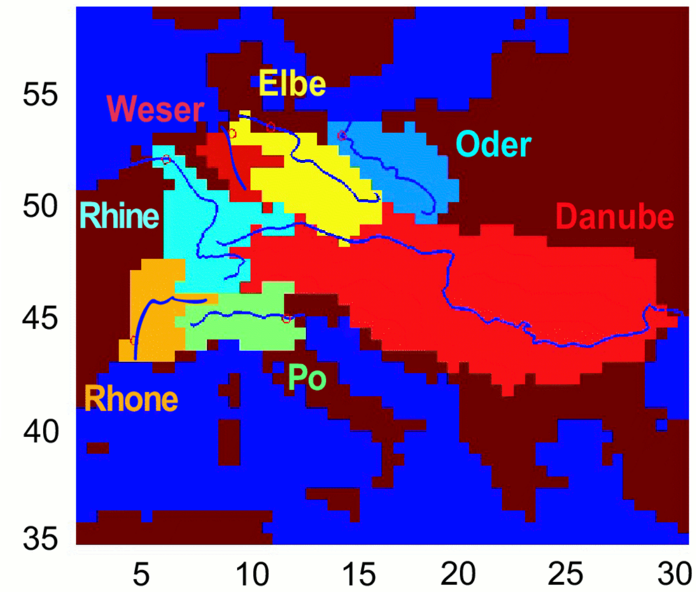
$$d_{j,k}(t) = \sum_{l=0}^{m_i-1} d_{j,k;i,l}\ \phi_{i,l}(t) \ .$$

- As 1-D base functions $\phi_{i,l}(t)$ we choose normalized endpoint-interpolating quadratic B-spline functions shown in the figures for (temporal) levels $i = 0, 1, 2$.



Schmidt et al., 2008

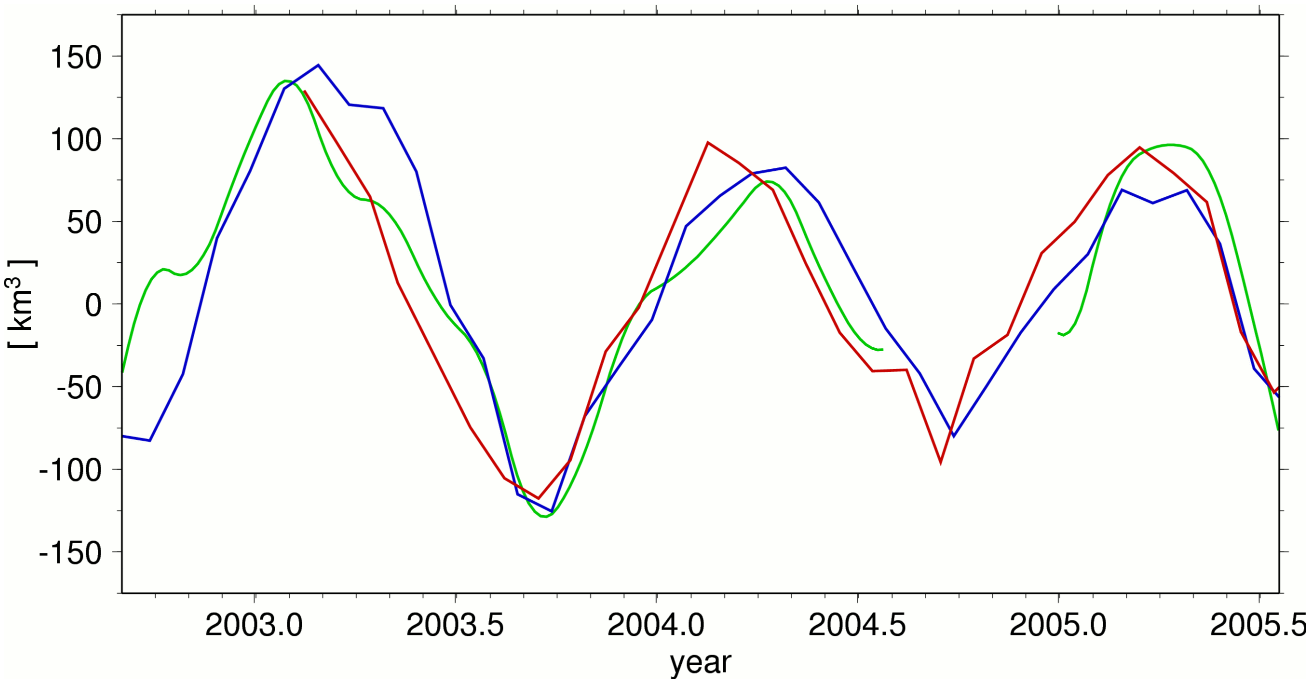
Example 1: water budget over Europe



regional Blackman/B-spline approach: $M_A(t)$

NCEP/gauge data: $S_A(t)$

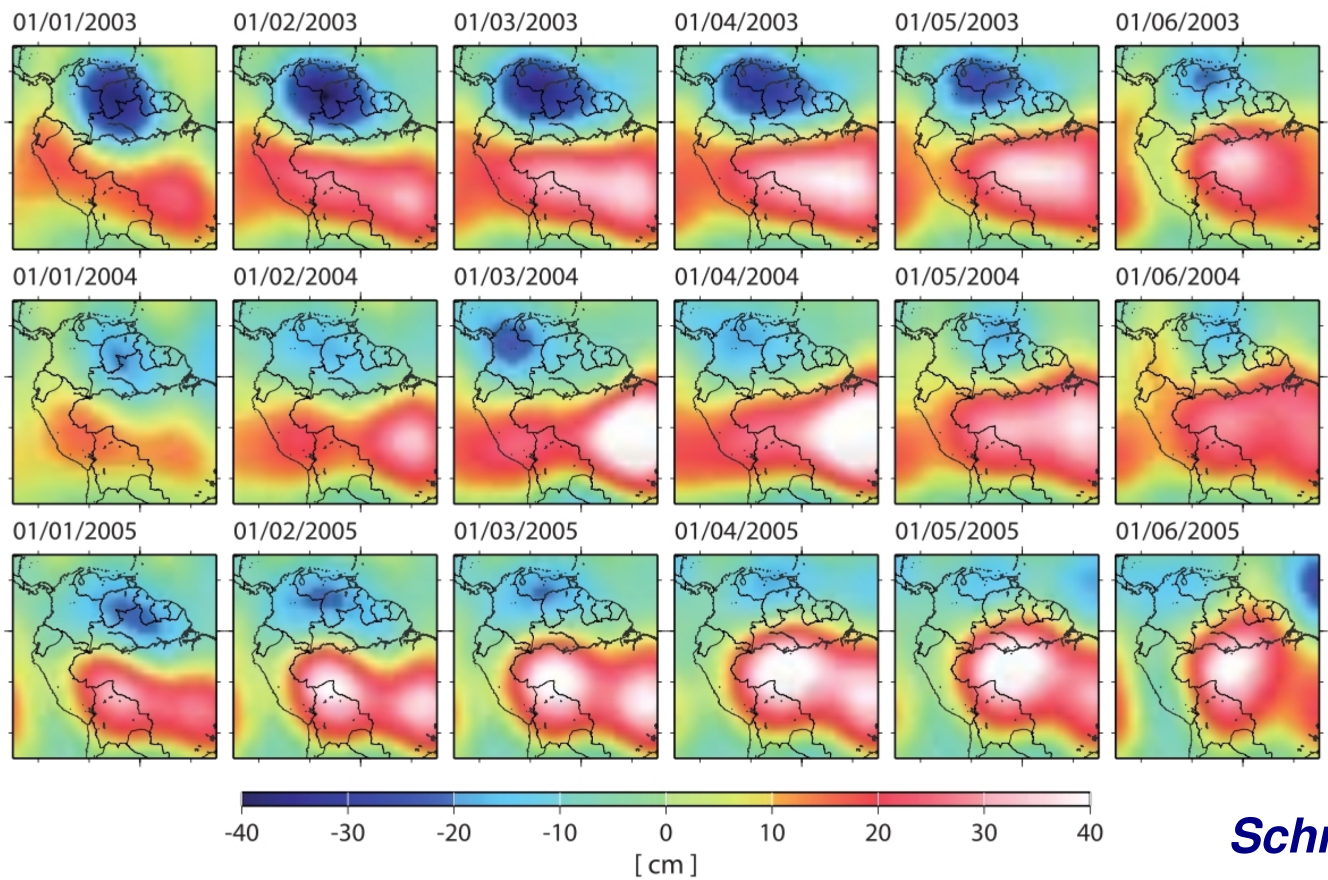
GFZ's RL04 solution



Seitz et al., 2009

- Apply a basin-mask averaging of the mass estimate.
- Validation: 4D multi-resolution estimates for spatial level 5 (SH deg. 40) compare well with L2 products and NCEP model.

Example 2: water storage variations (Amazon)



Schmidt et al., 2008

Figure 4. Level 4 EWHs $\delta h_4^{\text{ewh}}(\boldsymbol{r}, t_j)$ over the Amazon region. Since we set $i'' = 3$ in equation (29) the plots show, according to equation (9), signal parts until spherical harmonic degree $n = 23$ ($b = 2.2$).

- Signal components until SH degree 23
- A smooth, flexible 4D representation

4. Mascons

- SH expansion ΔA_{lm} of the gravity potential variations caused by a surface load $\sigma(t)$ at time t over a region *S*:

$$\Delta A_{\ell,m} = \frac{(1+k_\ell)R^2\sigma(t)}{(2\ell+1)M}\int_S Y_{\ell,m}(\Omega)d\Omega$$

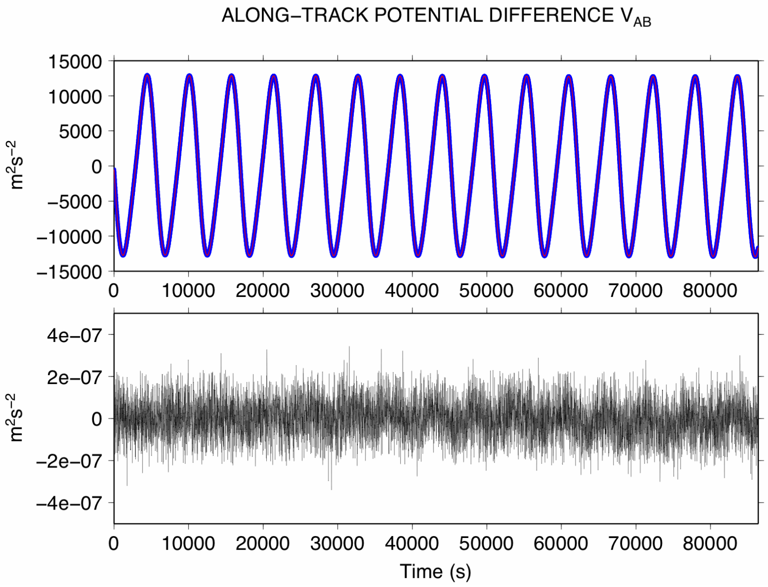
Loading Love number

Integration over the area S

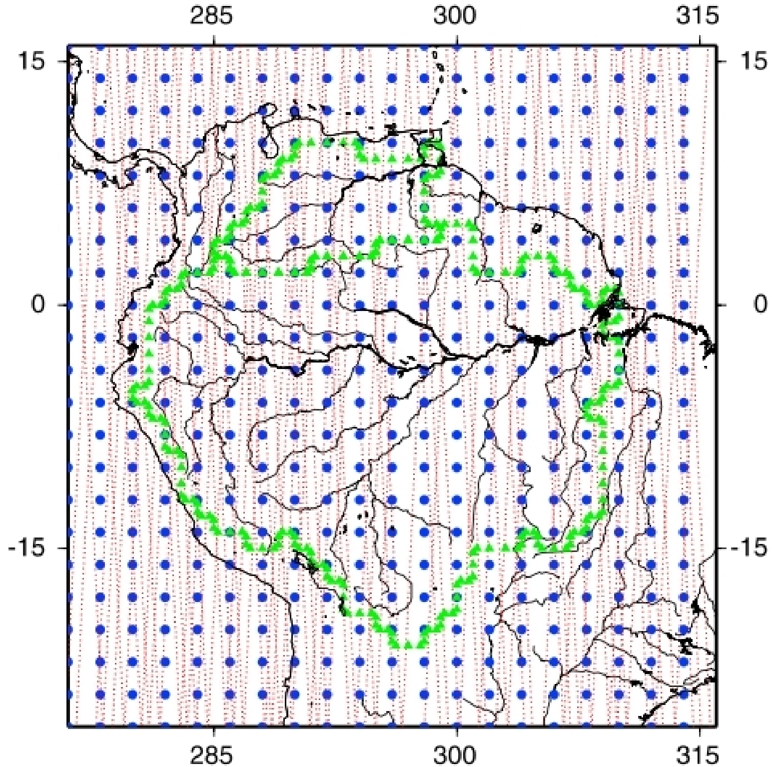
- The observation equations with respect to the mascons coefficients $\sigma(t)$ are thus a linear combination of the observation equations with respect to the global spherical harmonics coefficients $C_{lm}(t)$, $S_{lm}(t)$.
- An equivalent representation of the gravity field using a single layer potential. Local basis functions are the potentials of the regions S.
- Local regularization of the inversion : neighbour-type constraints between mascons, in space and/or in time domains (exponential weighting functions, ...).

Mascons

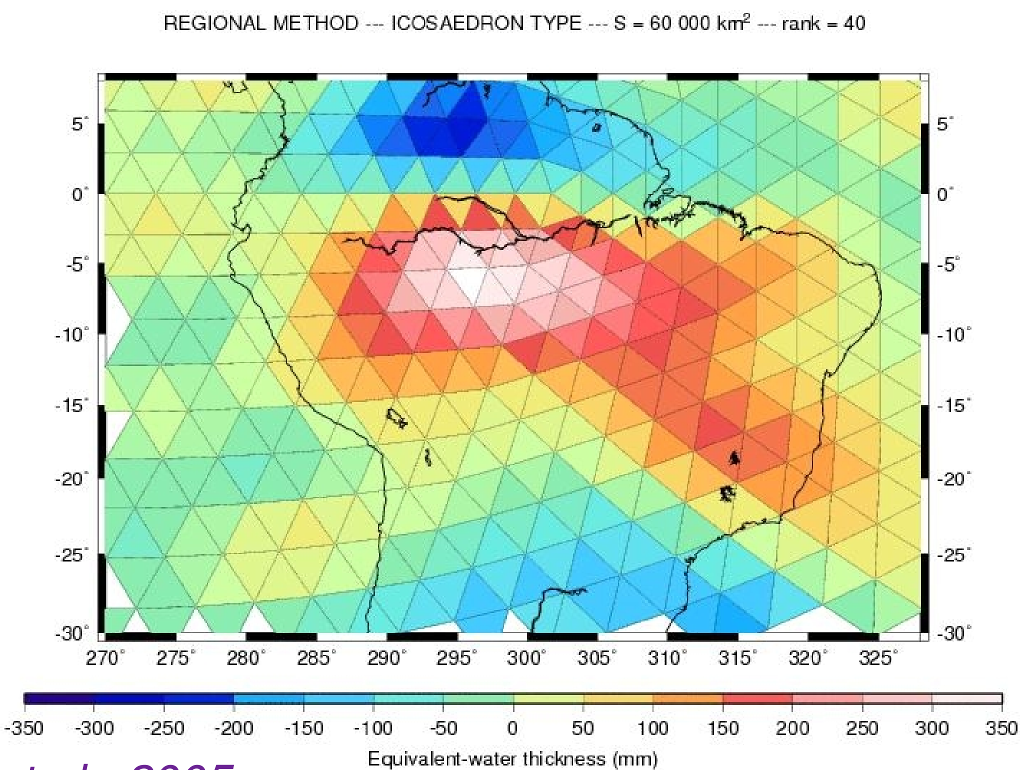
- Point masses, flat disks, spherical caps, spherical rings, triangles. Equal or variable areas.
- SH expansion of the mascon not always used
- Possible approximations for loading estimates (may impact on estimations of the water mass).



Ramillien et al., 2008

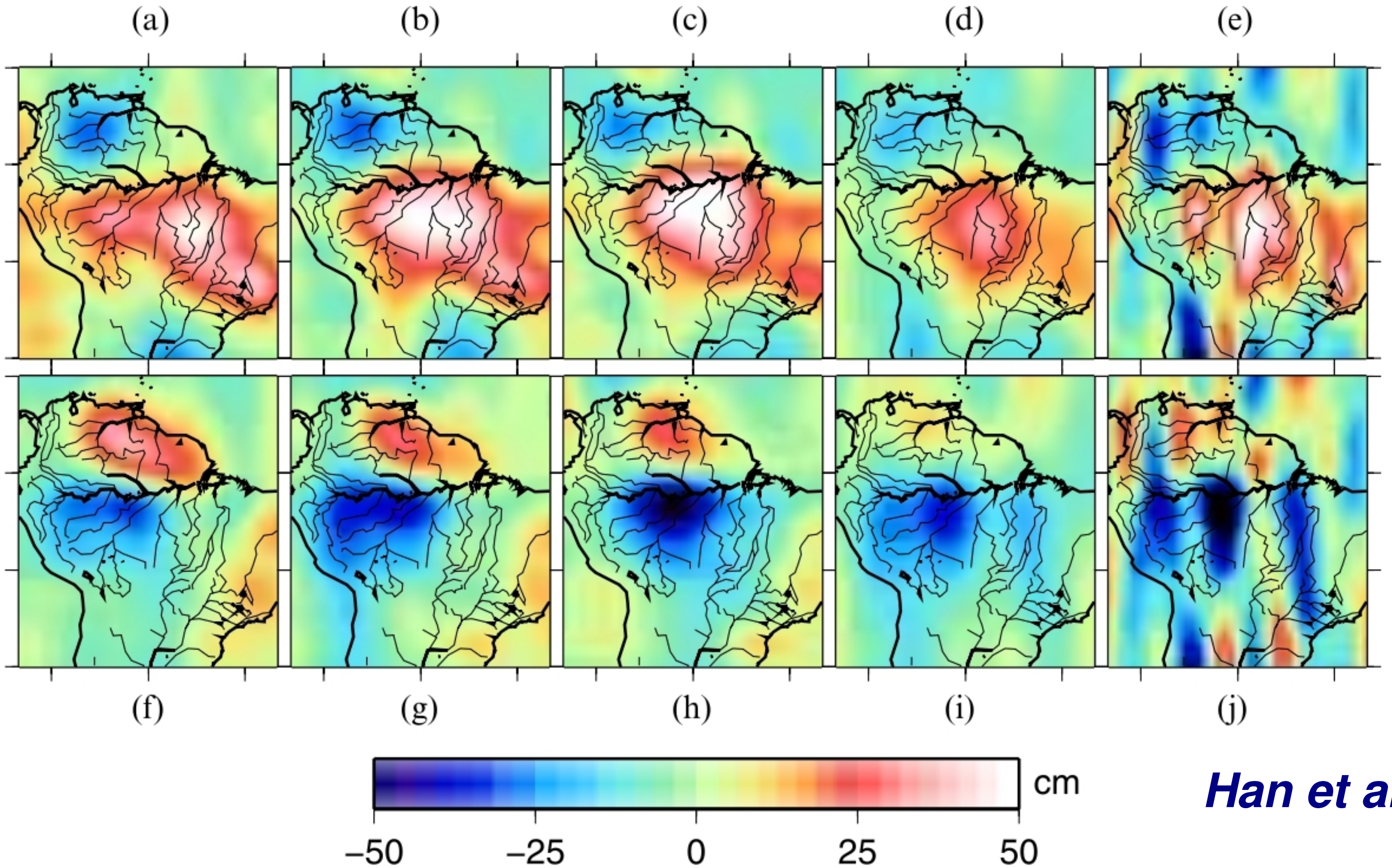


Han et al., 2005



See also: Lemoine et al., 2005, ...

Example: water storage variations



Han et al., 2008

Figure 8. Water storage variation in South America estimated from the localized analysis of RRR data every 15-day interval; first half of March (a), second half of March (b), and first half of April (c) in year 2005. The estimates from the monthly mean global harmonic solutions in March of 2005, (d) and (e), after the Gaussian smoothing with 500 km and 300 km averaging radius, respectively. (f) through (j) are the same as (a) through (e) but 6 month later.

- Enhancement of the water mass signal with respect to SH solutions filtered with isotropic gaussian filter, high temporal resolution.

Regional approaches: a comparison

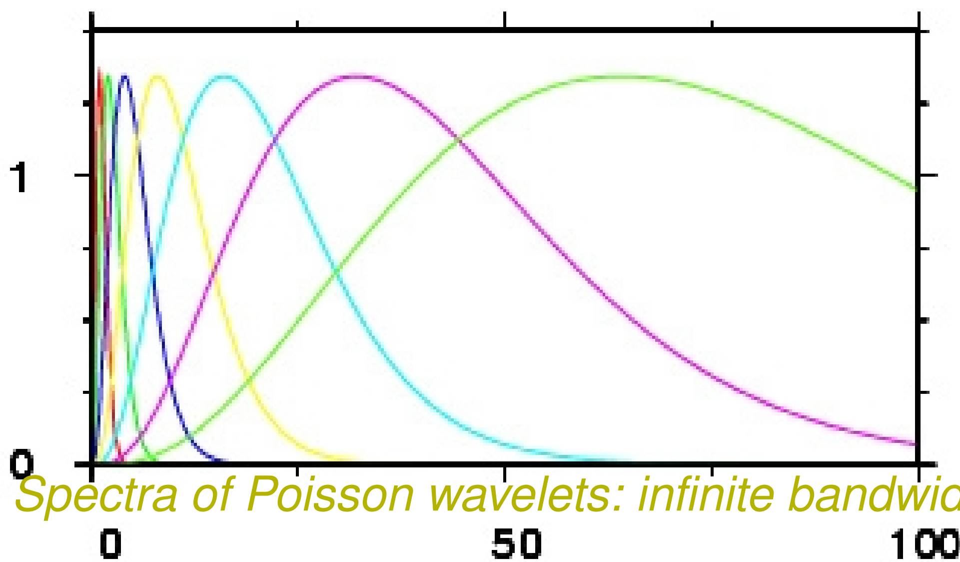
Approach	Characteristics	Applications
Slepian functions	⊥ - Optimal spatial or spectral concentration over a domain Very close to SH Uniform resolution over the area	Polar data gap Local signal modelling
Wavelets	Spatial & spectral localization: MRR analyses 4D: trade-off spatial/temporal resolution Equivalent regional sources (3D) Subdomain parameterization Hybrid SH/wavelet models	Local refinements (zoom-in) Data combination Local geophysical signal extraction
Splines	Interpolation function designed from the *a-priori* variance of the modelled signal Block parameterization Hybrid SH/spline models	Local refinements (zoom-in) Data combination
Mascons	Equivalent regional sources (single layer) + Fourier series for the temporal part. Neighbour-type constraints Loading approximations	Basin water mass estimates

The regularization

- The need for regularization depends on the parameterization of the local basis
- The functions selection is a regularization itself.
  - Example: smallest wavelet scale, grid density, spline shape
  - Example: mascons size
  - Orthogonal Slepian basis best conditioned
- If there are too few functions: the solution is too smooth
- If there are too many functions: regularization is needed.
  - The applied regularization can be viewed as a proper post-processing filtering (Klees *et al*., 2008 ; Kusche *et al*., 2008)
  - It may be locally tailored.

Signal enhancement

- Mass variations features vary significantly from one area to another. To fully take them into account in an SH model requires a high enough maximum degree. This may destabilize the global computation.
- Local basis functions may be locally added if needed. Their spectra may also be of infinite support. Finally, the regularization may be tailored for the area. This allow to extract more information from the data. Resolution obtained is controlled by local function resolution and regularization applied.



Spectra of Poisson wavelets: infinite bandwidth

- In the case of a GRACE FO mission configuration increasing the ill-posedness of the downward continuation operator, such approaches may be of particular interest.

Conclusion: What kind of regional products ?

- Different applications may benefit from different modelling approaches:
  - Different parameterization
  - Different regularization

Tailored local optimizations

- Thus, many possible L2 products !
  - In order to compare with geophysical models, the dampening related to the regularization must be precisely described in any case. Indeed the regularization plays an important role in the solution characteristics.
  - Long period signal: mitigate the loss or demonstrate that there is no loss of signal, interest of hybrid SH/local models.

- Users-defined regularization/parameterization requires their manipulation of Level 1b products.
  - Applications for local data combination & regional enhancements
  - Interest for sources separation: geophysical inversions may be better constrained in the case of a GRACE FO multi-dimensional sampling.